1. Carothers 3.18

2. Carothers 3.23

3. (Young's Inequality) Let $p \in (1, \infty)$ and define q by $\frac{1}{p} + \frac{1}{q} = 1$. Suppose $a, b \geq 0$. Show

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}$$

and that the inequality is strict unless either $a^{p-1} = b$ or $b^{q-1} = a$ (in which case both of these equalities hold!).

Hint: If $a = 0$ or $b = 0$ the result is obvious. Fix $b > 0$ and consider $f(a) = a^p/p + b^q/q - ab$ on $(0, \infty)$. Your job is to show $f(a) \geq 0$ with equality if and only if $a^p = b$. Sounds like an optimization problem! Look at the first and second derivatives of f.

Remark: You proof should clearly note the place where $p > 1$ is used.

4. Carothers 3.34

5. Carothers 3.36

6. Carothers 3.39

7. Carothers 3.46

8. Carothers 4.3

9. Carothers 4.11