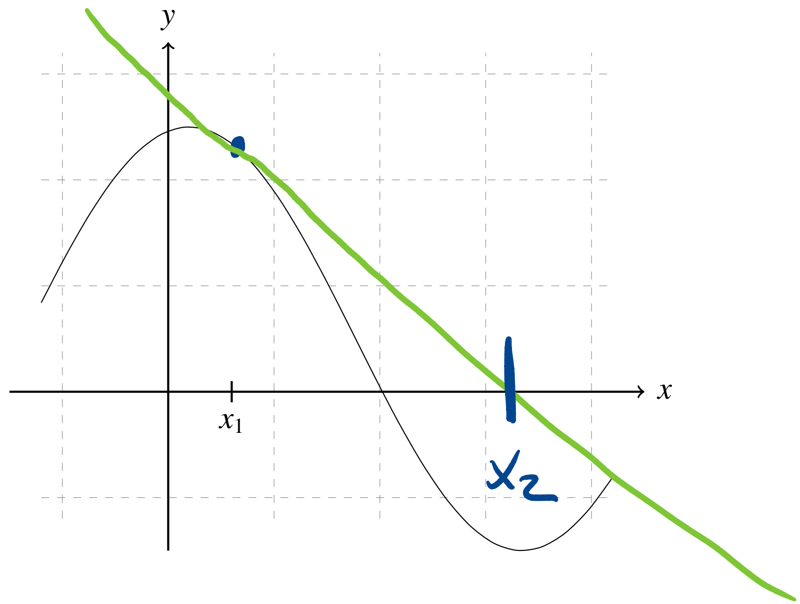
11. (10 points)

a. A generic graph $y = f(x)$ is shown and a first approximation x_1 is indicated. Show, by adding to the sketch, how Newton's method would find the next approximation x_2.



$f(x)$

b. For the equation $\underbrace{x^3 - 4x + 2}_{f(x)} = 0$ and the value $x_1 = -2$, compute x_2 from Newton's method.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f(x_1) = (-2)^3 - 4(-2) + 2$$
$$= -8 + 8 + 2$$
$$= 2$$

$$x_2 = (-2) - \frac{2}{8}$$
$$= -2 - \frac{1}{4} = -\frac{9}{4}$$

$$f'(x) = 3x^2 - 4$$
$$f'(-2) = 3 \cdot (-2)^2 - 4$$
$$= 12 - 4$$
$$= 8$$

12. (Extra Credit: 5 points)

Find **and simplify** the derivative of the function:

$$h(x) = \int_1^{e^x} \ln t \, dt$$

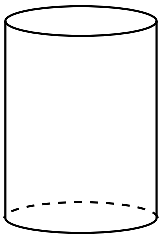
Explain your steps.

$$G(x) = \int_1^x \ln(t)\, dt$$

$$G(e^x)$$

4. (10 points)

The height of a right circular cylinder is increasing at rate of 3 meters per second while its volume remains constant. (See figure below.) At what rate is the radius changing when the radius and height are both 10 meters?



$V = 1000\pi$

$V = \pi r^2 h$

$$\frac{dV}{dt} = \pi\left[2r\frac{dr}{dt}\cdot h + r^2\frac{dh}{dt}\right]$$

$$0 = \pi\left[2\cdot 10\cdot\frac{dr}{dt}\cdot 10 + 10^2\cdot 3\right]$$

$$0 = 2\pi\frac{dr}{dt} + 3\pi$$

$$\frac{dr}{dt} = -\frac{3\pi}{2\pi} = -\frac{3}{2}\ \boxed{m/s}$$

Math 251: Final Exam

1. (10 points)

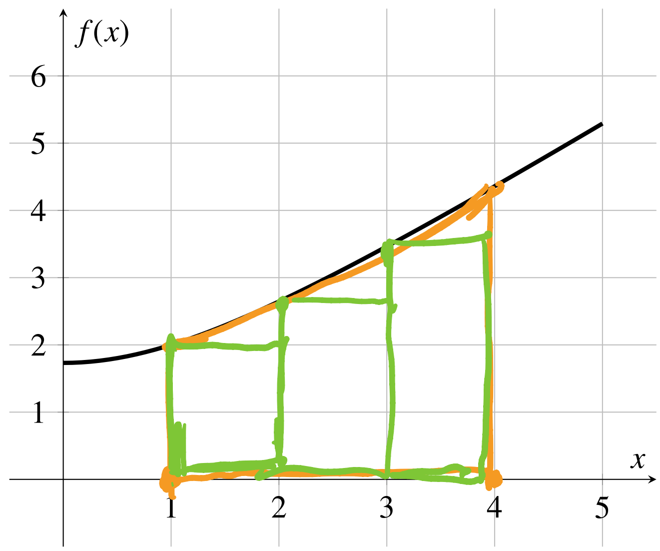
Find an equation of the tangent line to the curve at $x = e$: $\quad y = x^2 \ln x$

2. (10 points)

The graph of the function $f(x) = \sqrt{x^2+3}$ is shown.

a. On the graph sketch 3 rectangles, using left endpoints, that would be used to approximate

$$\int_1^4 \sqrt{x^2+3}\,dx.$$



b. Compute the approximation in part **(a)**. You do not need to simplify, but your answer should be in a form where a calculator would compute a numerical value.

$\Delta x = 1$

$f(1)\Delta x + f(2)\Delta x + f(3)\Delta x$

$(f(1) + f(2) + f(3))\Delta x$

$$\boxed{\left[\sqrt{1^2+3} + \sqrt{2^2+3} + \sqrt{3^2+3}\right]\cdot 1}$$

9. (10 points)

Short Answer

a. A population of chickadees is increasing at a rate of $r(t)$ chickadees per year. What does $\int_1^4 r(t)\,dt = 400$ mean? Make sure to include units in your answer.

The population of chickadees increased by 400 birds from time t=1 year to t=4 years (a three year time span).

b. Let $y = -3 + 5(x-4)$ be an equation of the tangent line to the graph of $f(x)$ at $x = 4$. Is it possible to determine $f(4)$ or $f'(4)$? Explain your answer.

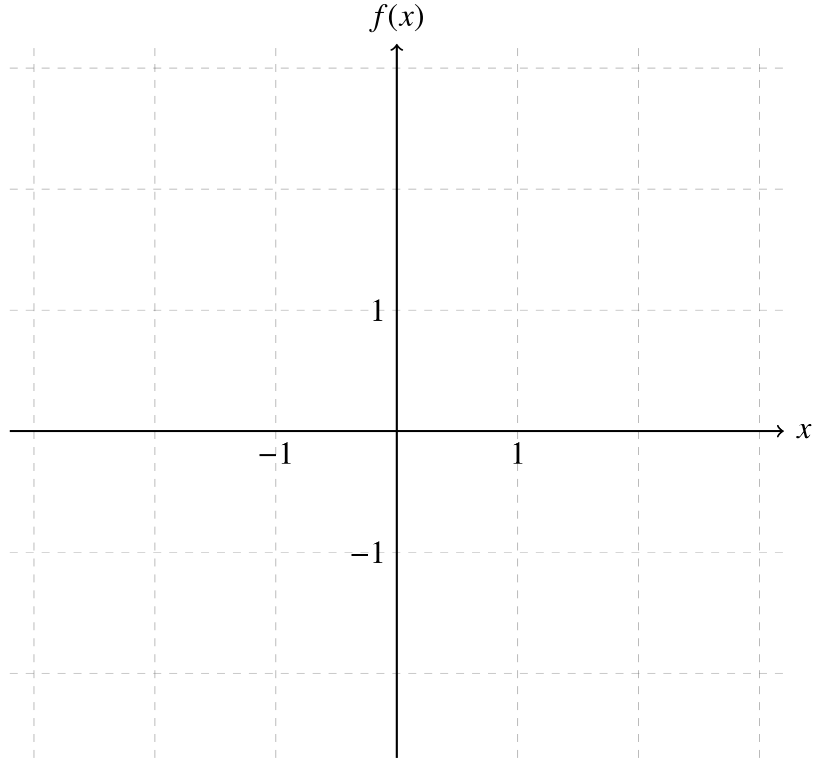
c. Let $C(T)$ be the number of chirps per second of a male cricket as a function of temperature, T, in degrees Fahrenheit. In the context of the problem, interpret $\underbrace{C'(70)} = 2$. Make sure to include units in your answer.

$\hookrightarrow [C'] = (\text{chirps}/\text{sec})/{}^\circ F$

If the ambient temp is 70°F the number of chirps per second increases at a rate of 2 chirps per second per °F. If the temperature rises to 71°F we expect the chirp rate to increase by 2 chirp per sec.

Problem 9 continued....

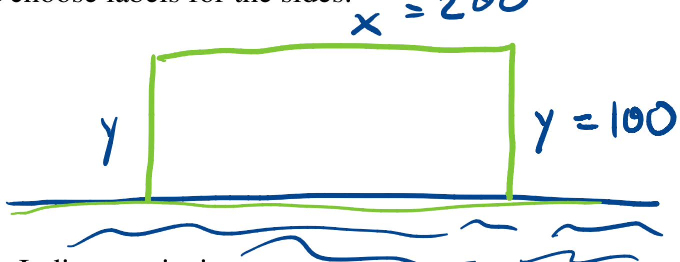
e. Sketch the graph on the axes:



10. (10 points)

A farmer has 400 meters of fencing and wants to fence off a rectangular field that borders a straight river. No fencing is needed along the river, which forms one side of the rectangle. What are the dimensions of the field that has the largest area?

a. Draw a sketch and choose labels for the sides.



b. Solve the problem. Indicate units in your answer.

Maximize area. $A = xy$

$2y + x = 400$

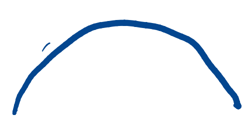
$x = 400 - 2y$

$A = (400-2y)\cdot y$

$= 400y - 2y^2$

$\boxed{0 \leq y \leq 200}$

$\frac{dA}{dy} = 400 - 4y$ $\frac{dA}{dy} = 0$ if $y = 100$



y	A
0	0
100	20000
200	0

$$A = 400 \cdot 100 - 2 \cdot 100 \cdot 100$$
$$= 40000 - 20000$$
$$= 20000$$

$x = 200\,m$

$y = 100\,m$

$$A = (400 - 2y)\,y$$

$$\frac{dA}{dy} = 400 - 4y \longrightarrow \quad y = 100 \Rightarrow \frac{dA}{dy} = 0$$

$$\frac{d^2A}{dy^2} = -4 < 0$$